

The RR Lyrae Period - K Luminosity relation for Globular Clusters: an observational approach^{*}

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ABSTRACT

The Period - metallicity - K band luminosity (PL_K) relation for RR Lyrae stars in 15 Galactic globular clusters and in the LMC globular cluster Reticulum has been derived. It is based on accurate near infrared (K) photometry combined with 2MASS and other literature data. The PL_K relation has been calibrated and compared with the previous empirical and theoretical determinations in literature. The zero point of the absolute calibration has been obtained from the K magnitude of RR Lyr whose distance modulus has been measured via trigonometric parallax with HST. Using this relation we obtain a distance modulus to the LMC of $(m - M)_0 = 18.54 \pm 0.15$ mag, in good agreement with recent determinations based on the analysis of Cepheid variable stars.

Key words: methods: observational – techniques: photometric – stars: distances – stars: variables: RR Lyrae – infrared: stars

1 INTRODUCTION

RR Lyrae, as well as classical Cepheids, are considered standard candles for estimating stellar distances in the Milky Way and to Local Group galaxies. They are produced in old stellar populations, and hence they provide important information for an understanding of the age, structure and formation of their parent stellar systems. As typical Population II stars, RR Lyrae are abundant in globular clusters and have been the subject of a huge number of studies for more than a century.

Although the pulsation theory explains quite well the connection between most of the involved physical quantities (van Albada & Baker 1971; Caputo, Marconi & Santolamazza 1998), the dependence of intrinsic luminosity on metal content has been the subject of debate for nearly three decades (see Smith 1995 for a review). Only recently the slope of the luminosity-metallicity relation $M_V - [Fe/H]$ seems to be converging towards a value ~ 0.20 - 0.23 that appears to be “universal” as supported by the most accurate studies of field RR Lyrae stars in the Milky Way (Fernley et al. 1998; Chaboyer 1999) and in the Large Magellanic Cloud (LMC, Gratton et al. 2004), and globular clusters in M31 (Rich et al. 2005).

Near infrared observations of variable stars present several advantages over optical investigations: a smaller dependence on interstellar extinction and metallicity, a smaller pulsational amplitude and more symmetrical light curves, and hence good mean magnitudes. In the last 20 years the calibration of the RR Lyrae period-IR luminosity relation (PL_K) has been the subject of several empirical and theoretical investigations (see Sect. 3), but there still remains some degree of uncertainty on the dependence of M_K on period and metallicity. To solve the problem of the dependence of M_K (RR) on these physical quantities, a large sample of RR Lyrae stars is needed, spanning a wide range of $[Fe/H]$, for which accurate K and $[Fe/H]$ measurements are available.

In this paper we present an accurate analysis of the infrared photometric properties of 538 RR Lyrae variables (376 RRab and 162 RRc) in 16 globular clusters (GC) with $-2.15 < [Fe/H] < -0.9$. This more than doubles the number of stars used in the previous largest study of this type (Nemec, Linnell Nemec & Lutz 1994). By means of such a large database we calibrated the PL_K relation constraining its dependence on period and metallicity on a strictly observational basis.

In §2 we describe the sample of RR Lyrae stars and the used photometric IR datasets. §3 is devoted to the description of the adopted method to calibrate the PL_K relation. In §4 we use the derived PL_K relation to estimate the distance to the calibrator GCs and to a sample of field RR Lyrae stars in the LMC, and compare the results with the previ-

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ous determinations in literature. Finally, we summarize our results in §5.

2 OBSERVATIONS

The data used in the present analysis consist of K photometry for the central regions of 9 GCs (M4, M5, M15, M55, M68, M92, M107 and ω Cen) derived from a set of images secured at the Telescopio Nazionale Galileo (TNG, Canary Islands), using the near-IR cameras ARNICA, and at the European Southern Observatory (ESO, La Silla), using the near-IR camera IRAC-2 and SOFI. A detailed description of the data reduction and calibration procedure can be found in Ferraro et al. (2000), Valenti et al. (2004), Valenti, Ferraro & Origlia (2004) and Sollima et al. (2004). All measured instrumental magnitudes were transformed into the Two-Micron All-Sky Survey (2MASS)¹ photometric system.

To extend our analysis to the outer regions of these clusters, we correlated our catalogs with the database obtained by 2MASS that extends to a wide area up to 15' from the cluster centers. The 2MASS K photometry for 5 additional GCs (M22, NGC 3201, NGC 5897, NGC 6362 and NGC 6584) was also considered.

For each cluster we identified a large number of variable stars by cross-correlating the IR catalog with the most comprehensive catalog of GC variable stars available in literature (Clement et al. 2001). Since the adopted K magnitudes are the average of repeated exposures and the K light curves of RR Lyrae variables have a fairly sinusoidal low amplitude shape, we assumed our K magnitudes as the mean magnitudes of the identified variables. Note that the individual K values from different datasets generally agree within less than 0.1 mag. So, we assumed ± 0.1 mag as a plausible error to attach to the mean K magnitudes used in the following analysis.

In addition, we considered the K photometry of RR Lyrae stars taken by Longmore et al. (1990, for variables in the clusters M3, M4, M5, M15, M107 and NGC 3201), Storm et al. (2004, IC4499), Butler (2003, M3), Dall'Ora et al. (2004, Reticulum) and Del Principe et al. (2005, M92). All magnitudes were reported to the homogeneous photometric system of 2MASS using the transformation equations provided by Carpenter (2001).

Table 1 lists for each calibrator GC the metallicity (in the Carretta & Gratton 1997 scale) and the reddening coefficient E(B-V) from Ferraro et al. (1999) together with the number of RR Lyrae considered in the present analysis.

3 METHOD

Theoretical studies on RR Lyrae stars performed in the past indicated a relationship between (infrared) luminosity, metallicity and period of the form

$$M_K = \alpha \log P_F + \beta [Fe/H] + \gamma \quad (1)$$

¹ See Skrutskie et al. (2006), Explanatory Supplement to the 2MASS All Sky Data Release, <http://www.irsa.ipac.caltech.edu/cgi-bin/Gator/>

Table 1. The sample of calibrator GCs. Metallicities are in the CG scale.

Name	[Fe/H]	E(B-V)	# RR Lyrae	Ref.
M3	-1.34	0.01	28	L90, B03
M4	-1.19	0.36	31	L90, F00, 2M
M5	-1.10	0.035	86	L90, V04, 2M
M15	-2.15	0.09	52	L90, VFO04, 2M
M22	-1.53	0.38	17	2M
M55	-1.61	0.07	8	F00, 2M
M68	-1.95	0.04	23	F00, 2M
M92	-2.15	0.025	11	D05, 2M
M107	-0.87	0.33	22	L90, F00, 2M
IC4499	-1.26	0.24	24	St04
NGC 3201	-1.23	0.21	60	L90, 2M
NGC 5897	-1.59	0.08	5	2M
NGC 6362	-0.92	0.08	31	2M
NGC 6584	-1.30	0.11	32	2M
ω Cen	-1.60 ²	0.11	82	L90, So04, 2M
Reticulum	-1.48	0.03	32	D04

² Because of the well known metallicity spread among RR Lyrae stars in this cluster (Sollima et al. 2006 and reference therein), we took into account only the metal-poor ($[Fe/H] < -1.4$)

References: L90: Longmore et al. (1990); 2M: 2MASS Skrutskie et al. (2006); F00: Ferraro et al. (2000); B03: Butler (2003); V04: Valenti et al. (2004); VFO04: Valenti, Ferraro & Origlia (2004); St04: Storm (2004); So04: Sollima et al. (2004); D04: Dall'Ora et al. (2004); D05: Del Principe et al. (2005).

where P_F indicates the period of the variables pulsating in the fundamental mode (RRab-type). The apparent K magnitude can be obtained by adding to both sides of eq (1) the K distance modulus

$$K = \alpha \log P_F + \beta [Fe/H] + \gamma + (m - M)_K \quad (2)$$

Several authors have calibrated PL_K relations, in the form of eq. (1), on the basis of observations in GCs and field RR Lyrae stars. In Table 2 the obtained empirical and theoretical PL_K relations are summarized. All the empirical relations use metallicities in the Zinn & West (1984) metallicity scale. Note that while the dependence on the period is quite consistent between the empirical and the theoretical values, the dependence on metallicity is about 3 times larger in theoretical estimates.

In the following sections we describe the adopted method to estimate the coefficients α, β and γ of eq. (1) using only observational constraints.

3.1 Dependence on Period (α)

The advantage of using GCs in constraining the coefficients α, β and γ lies in the fact that all the stars in a given cluster are at the same distance, and can be considered to share the same metal content and be subject to the same extinction effect. Therefore, within a given globular cluster eq. (2) can be written in the form

$$K - \alpha \log P_F = C \quad (3)$$

where C contains the information on reddening, metallicity and distance for that cluster. As a first step we estimate the constant C for each calibrator GC by assuming a first

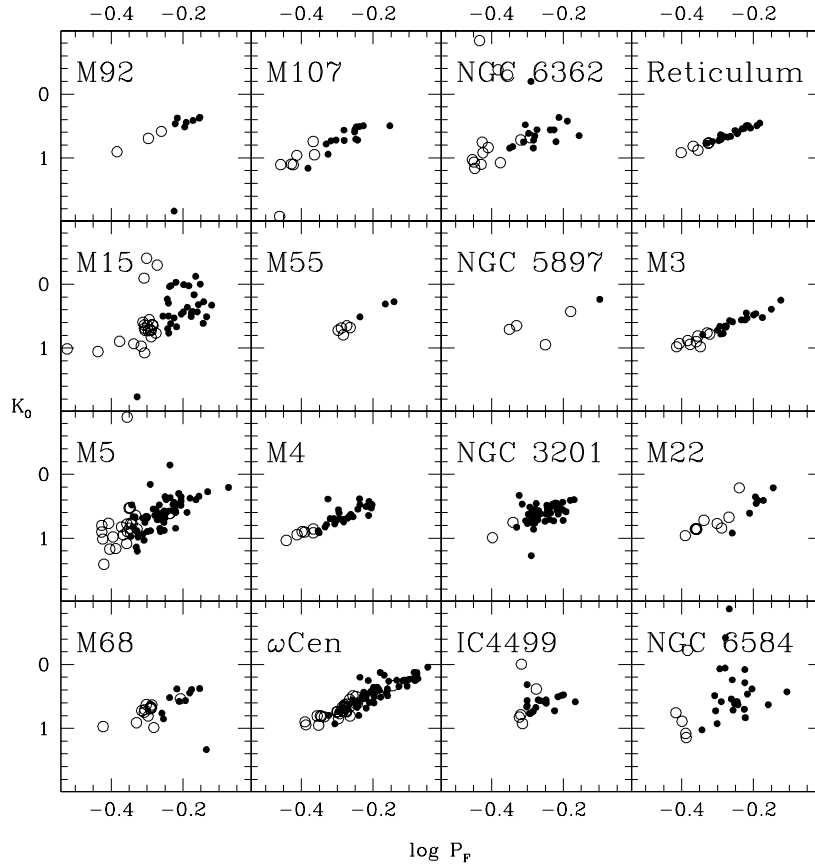


Figure 1. PL_K relation for the RR Lyrae of our calibrator GCs. Filled circles are the RRab variables, open circles are the RRc variables whose periods have been fundamentalized. K magnitude were scaled to the same reference distance and metallicity (see Sect. 3.1).

Table 2. Determinations of PL_K relations from previous studies.

Ref.	α	β	γ
Empirical			
J88	-1.72		-0.74
L90 (based on Jones et al. 1988)	-1.72	0.04	-0.65
L90 (based on Liu & Janes 1990)	-2.26	0.08	-0.76
LJ90	-2.72		-0.99
J92	-2.03	0.06	-0.72
J92	-2.33		-0.88
C92	-2.33		-0.88
S93	-2.95		-1.07
N94	-2.40	0.06	-1.27
N94	-2.40		-0.95
FS98	-2.34		-0.88
Theoretical			
B01	-2.071	0.167	-0.77
B03	-2.101	0.231	-0.77
C04	-2.353	0.175	-0.89

References: J88: Jones, Carney & Latham (1988); L90: Longmore et al. (1990); LJ90: Liu & Janes (1990); J92: Jones et al. (1992); C92: Carney, Storm & Jones (1992); S93: Skillen et al. (1993); N94: Nemec et al. (1994); FS98: Frolov & Samus (1998); B01: Bono et al. (2001); B03: Bono et al. (2003); C04: Catelan, Pritzl & Smith (2004).

guess value $\alpha = -2.4$ (Nemec et al. 1994) and averaging the left-hand side of eq. (3) for all the cluster RR Lyrae stars. First overtone RR Lyrae (RRc) are included in the analysis after correcting their periods by adding a constant term ($\Delta \log P = 0.127$). Outliers are identified as those stars whose C values differ from the median more than 3 times the semi-interquartile range² and rejected. Then, we scale the apparent K magnitudes for the corresponding value of C for each sample of GC RR Lyrae, merge all the samples together and best-fit eq. (3) by ordinary least squares thus estimating an updated value of α . An iterative procedure is carried out until a stable value of α is obtained. In Fig. 1 the distribution of the RR Lyrae scaled K magnitudes ($K_0 = K - C$) as a function of periods is shown for each of the 16 calibrator GCs. The procedure described above converges in few iterations to a global value of $\alpha = -2.38 \pm 0.04$ (see Fig. 2) and is insensitive to the first guess of α . The value of α estimated above is in good agreement with both empirical and theoretical estimates obtained in previous studies (see Table 2).

In Fig 3 we plot the individual slopes α_i calculated separately for each calibrator GC as a function of the clus-

² The interquartile range is defined as the distance between the 25th and the 75th percentile of the parent cumulative distribution of a given set of data.

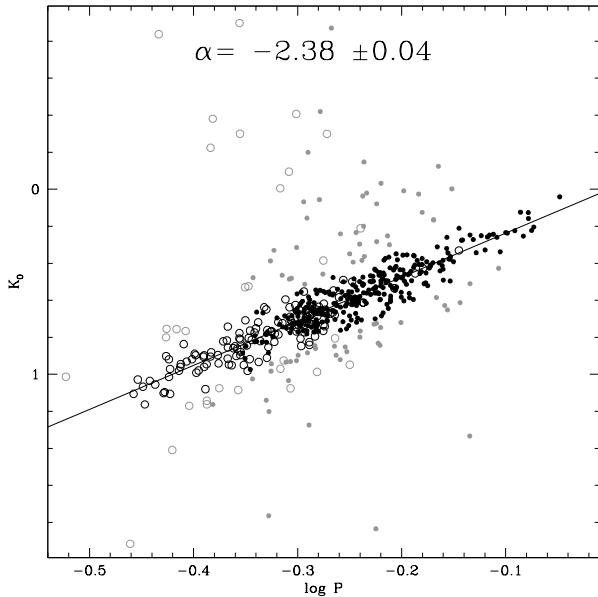


Figure 2. PL_K relation for the 538 RR Lyrae of our sample. The solid line represent the resulting fit. Filled circles are the RRab variables, open circles are the RRc variables whose periods have been fundamentalized. K magnitude were scaled to the same reference distance and metallicity (see Sect. 3.1). Grey symbols mark the points rejected to the fit.

ter metallicity. Note that, whereas an average value around $\alpha = -2.35$ is generally a good approximation for most clusters, α can significantly vary along the entire metallicity range. A Spearman rank correlation test gives a 78% probability that the two variables (α and $[Fe/H]$) are correlated between them. This effect is due to the complex interplay of dependences of the mass-luminosity, luminosity-temperature and K bolometric correction-temperature relations on metallicity that drives metal-rich variables to steeper PL_K relations. So, from this point of view the PL_K - $[Fe/H]$ relation is a bit less *universal* than we might wish.

3.2 Dependence on Metallicity (β)

Once the coefficient α is derived, two other parameters still remain to be derived in eq. (2) namely the slope $\beta = \delta K / \delta [Fe/H]$ and the zero point γ . Since both metallicity and distance vary from cluster to cluster, there is an evident degeneracy between the coefficient β and the additive term $\gamma + (m - M)_K$. No constraints can be assessed for β without an “a priori” knowledge on the distance and reddening of a subsample of our calibrator GCs. In order to disentangle this degeneracy in a homogeneous and completely observational way, we consider the distance determinations by Carretta et al. (2000) based on the application of the MS fitting technique to a sample of 12 GCs using Hipparcos trigonometric parallaxes of local subdwarfs. Table 3 lists the adopted distances and reddening coefficients for the 4 GCs (M5, M15, M68 and M92) in common between our sample and that of Carretta et al. (2000). Absolute distance moduli were converted to K band ones adopting the reddening coefficient

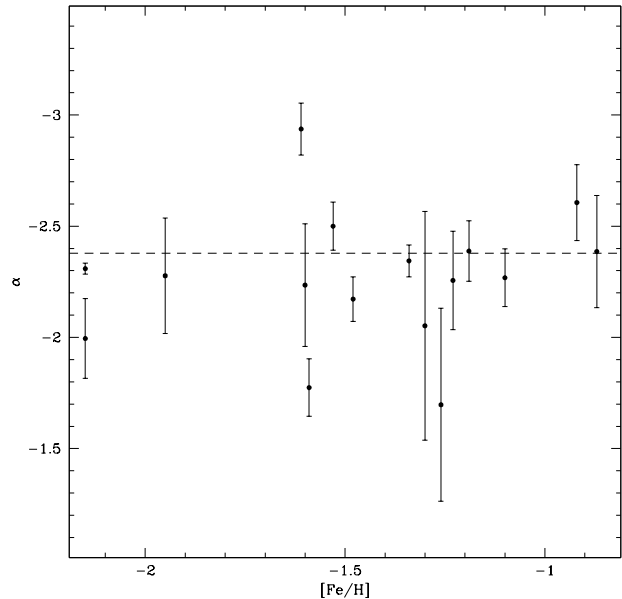


Figure 3. The parameter $\alpha = \delta K / \delta \log P$ for RR Lyrae stars in various globular clusters, as a function of the cluster metallicity. The dashed line indicate the general value of $\alpha = -2.38$ estimated for the entire sample.

Table 3. Distances and Reddening of the four distance calibrator GCs (from Carretta et al. 2000).

Name	[Fe/H]	$(m - M)_0$	E(B-V)
M5	-1.10	14.46	0.035
M15	-2.15	15.38	0.090
M68	-1.95	15.13	0.040
M92	-2.15	14.64	0.025

$A_K / E(B - V) = 0.38$ (Savage & Mathis 1979). Fig. 4 shows the bestfit of eq. (1). The resulting metallicity slope turns out to be $\beta = 0.08 \pm 0.11$ in full agreement with the values estimated in the past empirical studies listed in Table 2. The large uncertainty on the coefficient β reflects the sparse number of calibrator GCs. However, although such an uncertainty does not allow a conclusive sentence, our analysis confirms the discrepancy between the observed dependence of the PL_K relation on metallicity and that predicted by theoretical analyses.

3.3 Zero Point of the PL_K relation (γ)

The approach adopted in Sect. 3.2 allows to derive also the zero point of the PL_K relation (γ) directly by the bestfit of eq. (1) for the 4 GCs with known distances. However, although the relative distances of the GCs in the Carretta et al. (2000) sample are homogeneous between them, the zero point of that distance scale could be affected by systematic errors. As discussed by Gratton et al. (2003), the distance scale based on the MS fitting method could actually be ~ 0.1 mag fainter (i.e. longer) than that estimated by Carretta et al. (2000). For this reason we decided to calibrate our

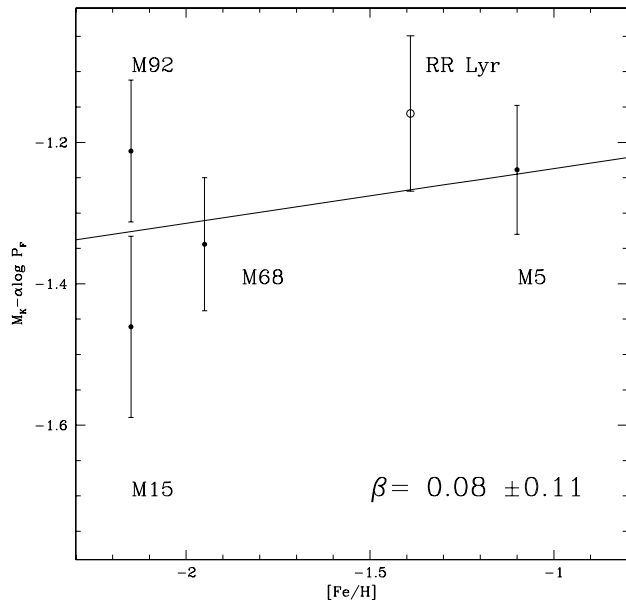


Figure 4. Bestfit of eq. (1) for the four calibrator GCs with known distances from Carretta et al. (2000). The open dot indicate the location in the diagram of the variable RR Lyr.

PL_K relation using the K magnitude of the variable RR Lyr, whose trigonometric parallax has been recently determined with HST (Benedict et al. 2002). We adopt for RR Lyr the period $P=0.5668054$ (Kazarovets, Samus & Durlevich 2001), the reddening coefficient $E(B - V) = 0.02$ (Benedict et al. 2002), the metallicity $[Fe/H] = -1.39$ (Clementini et al. 1995) and the distance modulus $(m - M)_0 = 7.08 \pm 0.11$ (Benedict et al. 2002). We use the value $K_{2MASS} = 6.52$ as the average of two measures taken half a cycle apart (Fernley, Skillen & Burki 1993). Given the relatively small amplitude and with the additional help of the templates by Jones, Carney & Fulbright (1996), we assumed this average to be reasonably close to the mean magnitude. Some additional uncertainty is given by the Blazhko effect that modulates the shape of the light curve with a double periodicity of 41 days and 4 years (Detre & Szeidl, 1973). Although the Blazhko modulation does not affect the intrinsic value of the mean magnitude, its estimate from two data points may be affected. So, in absence of a detailed and accurate K light curve for RR Lyr, we adopt the mean value $K=6.52$ (and hence $M_K=-0.57$), and we associate an error of ± 0.10 mag rather than the 1σ error of ~ 0.07 mag proposed by Fernley et al. (1993).

The location of RR Lyr in the $(M_K - \alpha \log P_F) - [Fe/H]$ plane is plotted in Fig. 4. As can be noted the absolute (period scaled) M_K magnitude of RR Lyr is 0.1 mag fainter than the value predicted by the bestfit of eq (1) for the 4 GCs listed in Table 3, in agreement with Gratton et al.'s (2003) considerations. Then, by constraining our PL_K relation to fit the absolute K magnitude of RR Lyr, we derive the zero point of the calibration, $\gamma = -1.05 \pm 0.13$. The uncertainty on the zero point is calculated by propagating the error on the parallax of RR Lyr with the resulting rms of the best-fit shown in Fig. 2.

Table 4. Derived distances to the calibrator GCs.

Name	$(m - M)_0$	α_i
M3	15.07	-2.34
M4	11.39	-2.39
M5	14.35	-2.27
M15	15.13	-1.99
M22	12.65	-2.50
M55	13.62	-2.94
M68	15.01	-2.28
M92	14.65	-2.31
M107	13.76	-2.39
IC4499	16.35	-1.70
NGC 3201	13.40	-2.26
NGC 5897	15.46	-1.77
NGC 6362	14.44	-2.61
NGC 6584	15.67	-2.05
ω Cen	13.72	-2.23
Reticulum	18.44	-2.17

As a result of this procedure, we obtain the following PL_K relations, in the Carretta & Gratton (1997) and Zinn & West (1984) metallicity scales, respectively:

$$M_K = -2.38(\pm 0.04) \log P_F + 0.08(\pm 0.11) [Fe/H]_{CG} - 1.05(\pm 0.13) \quad (4)$$

$$M_K = -2.38(\pm 0.04) \log P_F + 0.09(\pm 0.14) [Fe/H]_{ZW} - 1.04(\pm 0.13) \quad (5)$$

4 THE DISTANCE TO THE CALIBRATOR GCS AND TO THE LMC

We used such a relation to derive the distances to the calibrator GCs listed in Table 1. Distances were derived by bestfitting the eq. (4) to the RR Lyrae K magnitudes for each GC applying the outliers rejection procedure described in Sect. 3.1. The derived distance moduli are listed in Table 4.

As an independent check, we apply this procedure to the sample of RR Lyrae stars in the inner regions of the LMC whose metallicities (in the Zinn & West 1984 scale) and K magnitudes were derived by Borissova et al. (2004). The observed K magnitudes are corrected by -0.044 mag to convert them into the 2MASS photometric system according to Carpenter (2001). We adopt the reddening coefficient $E(B - V) = 0.11$ (Clementini et al. 2003). The resulting distance modulus to the LMC turns out to be $(m - M)_0 = 18.54 \pm 0.15$, in good agreement with the most recent infrared studies of Cepheid variables in the LMC (Persson et al., 2004; Gieren et al., 2005).

5 CONCLUSIONS

From an accurate analysis of 538 RR Lyrae variables in 16 GCs using infrared (K-band) photometry we derive a PL_K relation based on purely observational constraints. The derived dependences of the K magnitude on period and metallicity are in good agreement with those estimated by previous empirical studies. We confirm that the metallicity coefficient is about 2-3 times smaller than that predicted by

theoretical models, as it was found in all previous empirical analyses. The zero point of the calibration has been tied to the trigonometric parallax of RR Lyr measured with HST by Benedict et al. (2002). This calibration has been used to derive the distances to the 16 calibrator GCs considered in this analysis. As a further check, this relation has been applied to RR Lyrae stars in a few central fields in the LMC yielding a distance modulus $(m - M)_0 = 18.54 \pm 0.15$, in good agreement with the most recent determinations based on Cepheid variables (Persson et al., 2004; Gieren et al., 2005).

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REFERENCES

- Benedict G. F. et al., 2002, *ApJ*, 581, 115
 Bono G., Caputo F., Castellani V., Marconi M., 2001, *MNRAS*, 326, 1183
 Bono G., Caputo F., Castellani V., Marconi M., Storm J., Degl'Innocenti S., 2003, *MNRAS*, 344, 1097
 Borissova J., Minniti D., Rejkuba M., Alves D., Cook K. H., Freeman K. C., 2004, *A&A*, 423, 97
 Butler D. J., 2003, *A&A*, 405, 981
 Caputo F., Marconi M., Santolamazza P., 1998, *MNRAS*, 293, 364
 Carney B. W., Storm J., Jones R. V., 1992, *ApJ*, 386, 663
 Carpenter J. M., 2001, *AJ*, 121, 2851
 Carretta E., Gratton R. G., 1997, *A&AS*, 121, 95
 Carretta E., Gratton R. G., Clementini G., Fusi Pecci F., 2000, *ApJ*, 533, 215
 Catelan M., Pritzl B. J., Smith H. A., 2004, *ApJ Suppl. Ser.*, 154, 633
 Chaboyer B., 1999 in "Post-Hipparcos cosmic candles", Kluwer Ac. Pub., A. Heck, F. Caputo eds., 237, 111
 Clement C. et al., 2001, *AJ*, 122, 2587
 Clementini G., Carretta E., Gratton R., Merighi R., Mould J. R., McCarthy J. K., 1995, *AJ*, 110, 2319
 Clementini G., Gratton R., Bragaglia A., Carretta E., Di Fabrizio L., Maio M., 2003, *AJ*, 125, 1309
 Dall'Ora M. et al., 2004, *ApJ*, 610, 269
 Del Principe M., Piersimoni A. M., Bono G., Di Paola A., Dolci M., Marconi M., 2005, *AJ*, 109, 2714
 Detre L., Szeidl B., 1973, *IBVS*, 764
 Fernley J., Skillen I., Burki G., 1993, *A&AS*, 97, 815
 Fernley J., Skillen I., Carney B. W., Cacciari C., Janes K., 1998, *MNRAS*, 293, 61
 Ferraro F. R., Messineo M., Fusi Pecci F., De Palo M. A., Straniero O., Chieffi A., Limongi M., 1999 *AJ*, 118, 1738
 Ferraro F. R., Montegriffo P., Origlia L., Fusi Pecci F., 2000, *AJ*, 119, 1282
 Frolov M. S., Samus N. N., 1998, *AstL*, 24, 171
 Gieren W., Storm J. B., Thomas G. III, Fouqué P., Pietrzyński G., Kienle F., 2005, *ApJ*, 627, 224
 Gratton R. G., Bragaglia A., Clementini G., Carretta E., Di Fabrizio L., Maio M., Taribello E., 2004, *A&A*, 421, 937
 Gratton R. G., Bragaglia A., Carretta E., Clementini G., Desidera S., Grundahl F., Lucatello S., 2003, *A&A*, 408, 529
 Jones R. V., Carney B. W., Latham D. W., 1988, *ApJ*, 332, 206
 Jones R. V., Carney B. W., Storm J., Latham D. W., 1992, *ApJ*, 386, 646
 Jones R. V., Carney B. W., Fulbright J. P., 1996, *PASP*, 108, 877
 Kazarovets E. V., Samus N. N., Durlevich O. V., 2001, *Inf. Bull. Variable Stars*, 5135, 1
 Liu T., Janes K. A., 1990, *ApJ*, 354, 273
 Longmore A. J., Dixon R., Skillen I., Jameson R. F., Fernley J. A., 1990, *MNRAS*, 247, 684
 Nemec J. M., Linnell Nemec A. F., Lutz T. E., 1994, *AJ*, 108, 222
 Persson S. E., Madore B. F., Krzemiński W., Freedman W. L., Roth M., Murphy D. C., 2004, *AJ*, 128, 2239
 Rey S. C., Lee Y. W., Joo J. M., Walker A., Baird S., 2000, *AJ*, 119, 1824
 Rich R. M., Corsi C. E., Cacciari C., Federici L., Fusi Pecci F., Djorgovski S. G., Freedman W. L., 2005, *AJ*, 129, 2670
 Savage B. D., Mathis J. S., 1979, *ARA&A* 17, 73
 Skillen I., Fernley J. A., Stobie R. S., Jameson R. F., 1993, *MNRAS*, 265, 301
 Skrutskie M. F. et al., *AJ*, 131, 1163
 Smith H. A., 1990 in "RR Lyrae stars", Cambridge University Press
 Sollima A., Ferraro F. R., Origlia L., Pancino E., Bellazzini M., 2004, *A&A*, 420, 173
 Storm J., 2004, *A&A*, 415, 987
 Valenti E., Ferraro F. R., Perina S., Origlia L., 2004, *A&A*, 419, 139
 Valenti E., Ferraro F. R., Origlia L., 2004, *MNRAS*, 351, 1204
 van Albada T. S., Baker N., 1971, *ApJ*, 169, 311
 Zinn R., West M. J., 1984, *ApJS*, 55, 45